Novel techniques for identifying magnetohydrodynamic wave modes in sunspot umbrae in the DKIST era.

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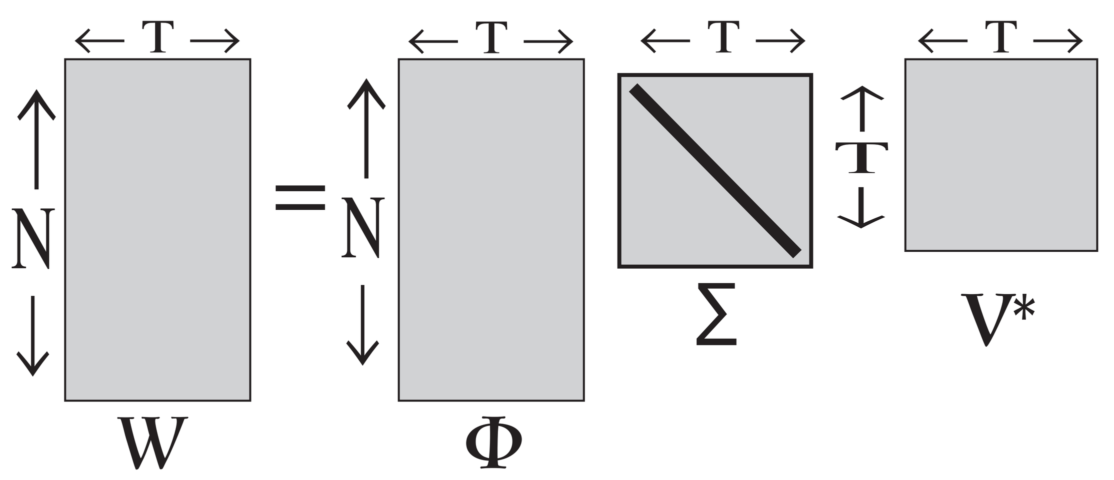
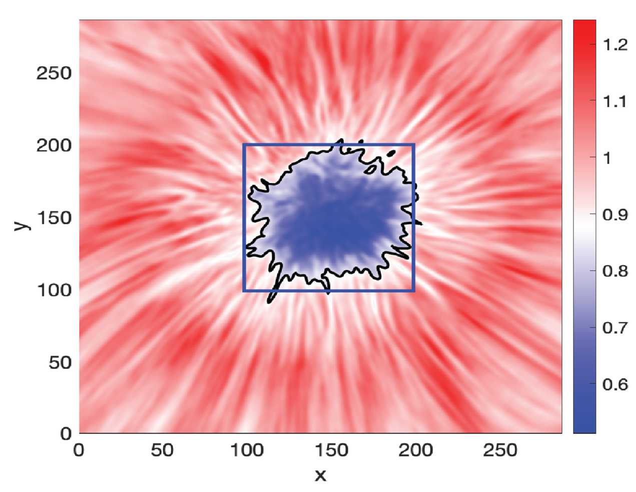
Introduction

High resolution observations of pores and sunspots show a rich and complex variety of oscillatory temporal and spatial behaviour. To decompose this data into individual magnetohydrodynamic (MHD) wave modes is non-trivial and requires a multi-faceted approach. We apply the proper orthogonal decomposition (POD) and Dynamic Mode decomposition (DMD) techniques to identify the dominant MHD wave modes in a sunspot using intensity time series. POD classifies modes that are orthogonal in space, but places no restrictions on their frequencies. DMD has no such spatial restrictions, but classifies modes that are orthogonal in time, i.e. the identified modes cannot have the same frequency. Here we show that the combined POD and DMD approaches can successfully identify both sausage and kink modes in a sunspot umbra with an approximately circular cross-sectional shape [1].

Decomposition techniques

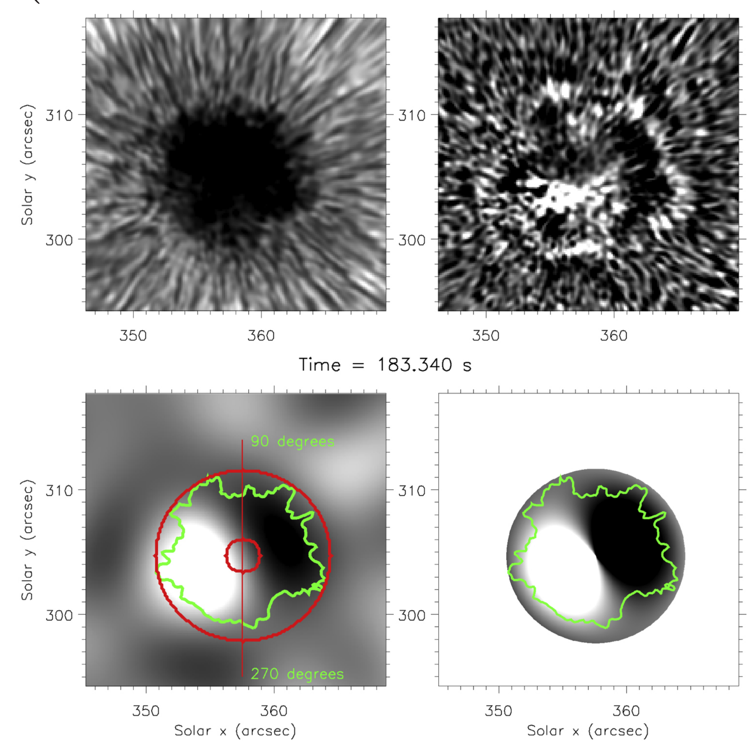
- The details of POD and DMD techniques have been described in details by [1]
- While POD looks for orthogonality in space, DMD looks for orthogonality in time.
- POD ranks the mode in terms of their contribution to the signal, while DMD does not.
- To apply POD and DMD, first we reorganise the snapshots as a column vector to build up the matrix W.
- The POD of W is obtained by the singular value decomposition (SVD), $W = \Phi\Sigma V^*$.
- For DMD, decompose the matrix W in two matrices, W^A and W^B, so that W^B is shifted by a snapshot relative to W^A. These matrices have a linear relationship, i.e. $W^B = AW^A$. The eigenvalues, u_i, and eigenvectors, v_i, of A, give the DMD modes and their frequency.





Sunspot case study

We use Hydrogen-alpha (Hα) observations with 0.138$''$ sampling per pixel (about 100 km per pixel) and time cadence of 1.78 s. Recently, Jess et al (2017) [2] used this data to identify kink modes rotating in the azimuthal direction (see below) by implementing a $k-\omega$ Fourier filter (0.45 − 0.90 arcsec^{-1} and 5 − 6.3 mHz).

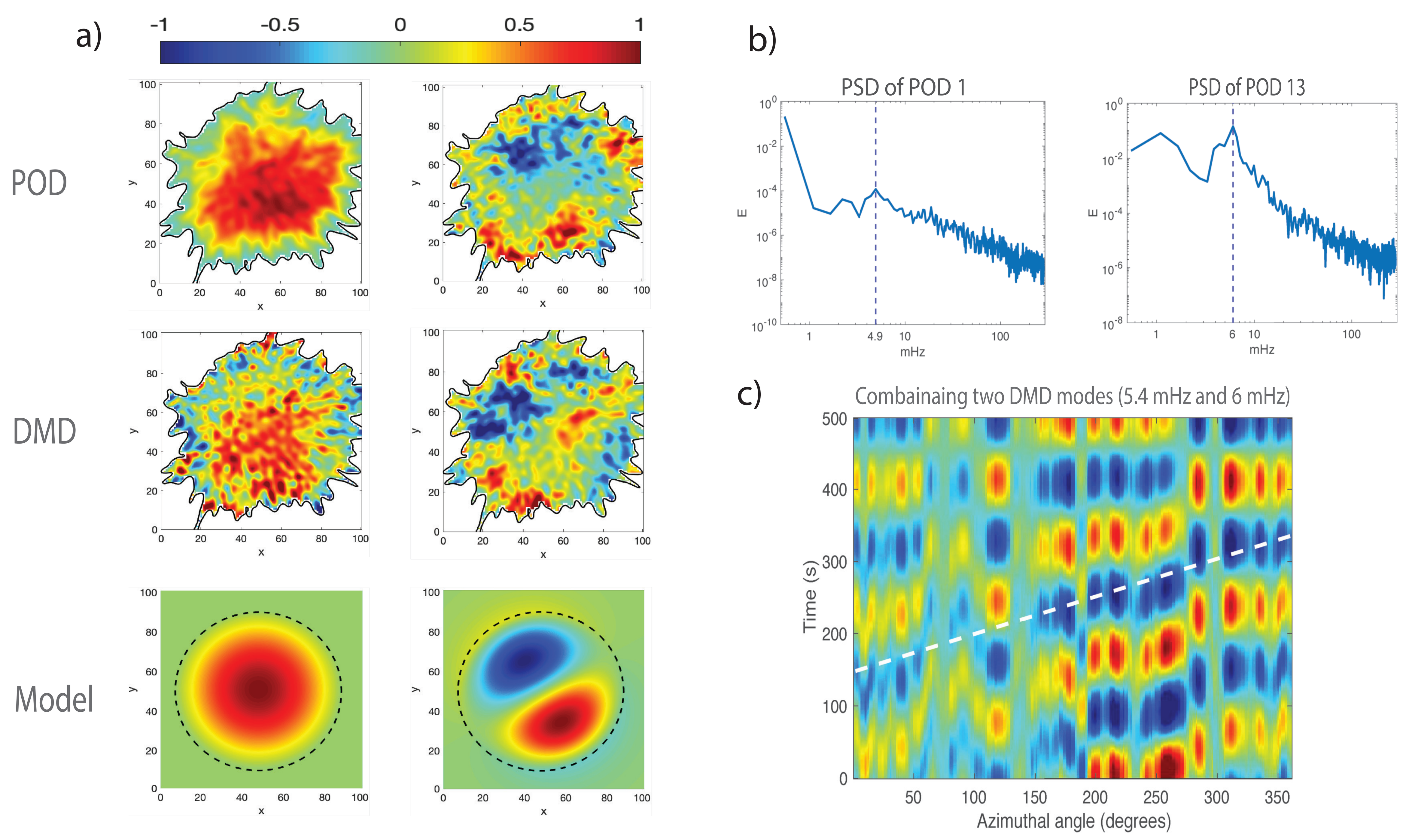


Applying POD and DMD

The POD and DMD analysis was applied to the particular Region of Interest (ROI) of the sunspot umbra as shown by the blue box in the above figure, and the results are shown on the right figure, where:

panel (a) shows the detection of the fundamental slow body sausage mode (first column) and the fundamental slow body kink mode (second column). While the first row displays the POD modes, POD 1 (top left) and POD 13 (top right). The second row displays the DMD modes, DMD that correspond to 4.8 mHz (middle left) and DMD that correspond to 6 mHz (middle right). The third row shows the theoretical model.
panel (b) shows the power spectrum density (PSD) of the POD time coefficient.
panel (c) shows the time-azimuth diagram of the combination of two DMD (5.4 and 6 mHz) modes, showing the apparent rotational motion of the kink mode as detected by [2].

The observed modes



Conclusions and future work

The POD and DMD techniques are excellent tools for the identification of MHD waves in sunspot data. These techniques do not require filtering, making them powerful tools to identify higher-order modes in sunspot data, as filtering would completely skew the results.

Currently applying these techniques in conjunction with sunspots with an elliptical cross-sectional shape. Later, with a robust identification methodology established, we will apply the POD and DMD techniques on data obtained for an irregular cross-sectional shape.

These techniques will be indispensable for mode identification in high-resolution data provided by DKIST.

References

[1] A. Albidah *et al.* Proper orthogonal and dynamic mode decomposition of sunspot data. *Philosophical Transactions of the Royal Society A*, 379(2190):20200181, 2020.

[2] D. B. Jess *et al.* An inside look at sunspot oscillations with higher azimuthal wavenumbers. *The Astrophysical Journal*, 842(1):59, 2017.